On the Origin of the Internal Structure of Haloes

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ABSTRACT

High-resolution N-body simulations of hierarchical cosmologies have shown that the density and velocity dispersion profiles of dark-matter haloes display well-definite universal forms whose origin remains unknown. In the present paper, we calculate the internal structure of haloes expected to arise in any such cosmologies by simply taking into account that halo growth proceeds through an alternate sequence of discrete major mergers and long periods of gentle accretion. Major mergers cause the violent relaxation of the system subject to the boundary conditions imposed by accreting layers beginning to fall in at that moment. Accretion makes the system develop inside-out from the previous seed according to the spherical infall model. The predicted structure is in very good agreement with the results of numerical simulations, particularly for moderate and low mass haloes. We find strong indications that the slight departure observed in more massive systems is not due to a poorer theoretical prediction, but to the more marked effects of the limited resolution used in the simulations on the empirical profiles. This may have important consequences on the reported universality of halo structure.

Key words: cosmology – dark matter – galaxies: formation, evolution

1 INTRODUCTION

The dominant dark component of matter in the universe appears to be clustered in bound haloes which form the skeleton of all astronomical objects of cosmological interest, from dwarf galaxies to rich galaxy clusters. All we know about these systems comes from their gravitational effects on the luminous matter they trap or on the light traveling across them. This is so little information that the following fundamental questions have prevailed for a long time. Does the internal structure of haloes depend on their mass? Does it depend on their past history? And on cosmology? How is this structure set?

Gunn & Gott (1972; Gott 1975; Gunn 1977) were the first to address this problem by considering the simplified case of haloes forming through spherical infall, that is, the collapse of a density fluctuation of smooth, isotropically distributed, dissipationless matter in an otherwise homogeneous expanding universe. Under the adiabatic invariant collapse approximation, they derived the density profile arising from some specific initial conditions. The effects of refining the derivation used and of adopting more and more realistic initial conditions have been subsequently addressed in a series of works (Fillmore & Goldreich 1984; Bertschinger 1985; Hoffman & Shaham 1985; Ryden & Gunn 1987; Ryden 1988; Zaroubi & Hoffman 1993; Łokas 2000; Łokas &

Hoffman 2000; Del Popolo et al. 2000; Engineer, Kanekar & Padmanabhan 2000; Nusser 2001).

An important result of this research line is that haloes grow inside-out as new material is being incorporated in secondary infall. That is, despite the continuous shell-crossing (and radial relaxation, see below) of the infalling layers with the previously relaxed body, the structure remains at any moment essentially unaltered within the instantaneous radius. As shown below, this sole consideration should permit us to determine the halo density profile provided the infall rate of matter is known. Much progress has been achieved in the last twenty years in the modelling of halo growth. The extended Press-Schechter (PS) model (Press & Schechter 1974; Bower 1991; Bond et al. 1991; Lacey & Cole 1993) makes quite accurate predictions, indeed, on the rate at which haloes increase their mass (Lacey & Cole 1994) in hierarchical cosmologies like the one describing the real universe. Unfortunately, haloes develop in such cosmologies through continuous mergers rather than through smooth spherical infall.

The effects of mergers depend on the relative mass of the progenitors. Major mergers bring the whole system out of equilibrium making it move in the phase space around some attractor. During this process, particles experience random accelerations owing to the rapidly varying collective potential well which causes the relaxation of the system (Lynden-

Bell 1967) in a similar although more dramatic way than in the case of spherically infalling layers. This violent relaxation erases any imprint of the previous history of the system, the new equilibrium state reached being characterised by a normal distribution of particle velocities similar to that yielded by the common two-body relaxation, although independent of particle mass. Were haloes isolated and, hence, unperturbed after any such dramatic event they would end up as infinite, spherical systems with a uniform, monomass, isotropic velocity dispersion. This is the reason why haloes are often modelled as isotropic, isothermal spheres (e.g., King 1972; Shapiro, Iliev & Raga 1999). But haloes are not isolated systems. During the violent relaxation process, they keep on collecting matter through minor mergers making the boundary conditions to vary in some unknown way. This severely limits the predictive power of the violent relaxation theory (e.g., Saslaw 1987).

To gain a deeper insight on the internal structure of haloes many authors have turned to numerical experiments. Using high-resolution cosmological N-body simulations Navarro, Frenk & White (1996, 1997) have found that the spherically averaged density profile of haloes of all masses is well fitted by the simple analytical expression

$$\rho(r) = \frac{\rho_c r_s^3}{r(r_s + r)^2} \tag{1}$$

in any hierarchical cosmology analysed. In equation (1), r is the radial distance to the halo centre, and r_s and ρ_c are the halo scale radius and characteristic density, respectively. The latter two parameters are related to each other and to the mass M of the halo through the condition that the virial radius R of the system encloses, by definition, an average density equal to a fixed factor f times some density ρ_u characterising the universe at that moment. The values of f found in the literature are in the range [178, 500], while ρ_u is taken equal to ρ_{crit}, the critical density for closure, or $\bar\rho$, the mean cosmic density. Navarro and collaborators adopted $f = 200$ and $\rho_u = \rho_{crit}$.

On the other hand, the spherically averaged, locally isotropised, velocity dispersion profile $\Sigma(r)$ is found to be well fitted by the solution of the Jeans equation for hydrostatic equilibrium and negligible rotation,

$$\Sigma^2(r) \left(\frac{\mathrm{d}\ln\Sigma^2}{\mathrm{d}\ln r} + \frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r} \right) = -\frac{3\,GM(r)}{r} \tag{2}$$

with $\rho(r)$ the function given in equation (1) and $M(r)$ the corresponding mass within r, by adopting the boundary condition of null pressure at infinity. Note that, although the hydrostatic condition is natural to hold, the boundary condition at infinity is not obvious owing to the limited extent of haloes.

These empirical results have been confirmed by other authors (Cole & Lacey 1997; Huss, Jain & Steinmetz 1999b; Bullock et al. 2001), there being some controversy only at very small radii (Moore et al. 1998; Jing & Suto 2000; Fukushige & Makino 2001) where they are the most affected by the spatial resolution of the simulations. In any event, the fundamental question about the origin of that empirical halo structure remains. It is not even clear whether this is a general result of gravitational collapse, including smooth spherical infall, or the specific consequence of repeated mergers

(Huss, Jain & Steinmetz 1999a; Syer & White 1998; Moore et al. 1999; Łokas & Hoffman 2000).

In the present paper, we use a variant of the extended PS model distinguishing between minor and major mergers (§ 2) to derive the structure of haloes predicted in hierarchical cosmologies and compare it with that found in numerical simulations (§ 3). The result of this comparison supports the idea that the internal structure of haloes is the natural imprint of their hierarchical growth (§ 4).

2   MINOR AND MAJOR MERGERS

The scaling of the previous density profile varies with cosmology although the same trend is always found: the characteristic density ρ_c decreases with increasing halo mass. Navarro, Frenk & White (1997) suggested that this universal trend is due to the fact that, in hierarchical cosmologies, more massive haloes form later when the mean density of the universe is lower. Unfortunately, the idea that haloes fix their structural properties at formation is hard to assess because the own concepts of halo formation and destruction are rather fuzzy in hierarchical cosmologies in which haloes grow through continuous mergers.

To properly define these concepts it is convenient to distinguish between minor and major mergers (Manrique & Salvador-Solé 1996; Kitayama & Suto 1996; Salvador-Solé, Solanes & Manrique 1998; Percival, Miller & Peacock 2000; Cohn, Bagla & White 2001). In the Modified Press-Schechter (MPS) model (Salvador-Solé, Solanes & Manrique 1998; Raig, González-Casado & Salvador-Solé 1998; Raig, González-Casado & Salvador-Solé 2001) mergers producing a fractional mass increase above a given threshold $\Delta_m \sim 0.5$ are regarded as major, and minor otherwise. Note that a given merger may be seen as major or minor depending on the viewpoint of the partner halo which is considered. Since haloes are essentially unperturbed in minor mergers while they are completely rearranged in major ones we say that haloes *survive* in the former case and *are destroyed* in the latter. The destruction of a halo does not necessarily imply the formation of a new one: the largest partner partaking of the merger can see it as minor and, hence, survive to it being identified to the final system. This kind of mergers therefore correspond to the *accretion* by the most massive halo of the remaining partners. (The definitions of minor and major mergers given above imply that there is at most one surviving halo in any given merger.) Only those mergers in which *all* partners are destroyed or, equivalently, no merging halo survives, do mark the *formation* of new haloes.

The MPS model yields analytical expressions for the mass accretion rate of haloes (see eq. [3] below), their destruction and formation rates, as well as the distribution probability functions of progenitor masses and of formation and destruction times whose median values define, respectively, the typical mass of progenitors and the typical halo formation and destruction times. All these theoretical quantities are in good agreement with the results of N-body simulations (Raig, González-Casado & Salvador-Solé 2001).

In particular, the average rate at which mass is accreted onto a halo with M at t is, according to the definition of Δ_m,

$$r_{\rm a}(M,t) = \int_M^{M(1+\Delta_{\rm m})} (M' - M)\, r_{\rm t}(M,M',t)\, {\rm dM'} ,\qquad (3)$$

where

$$r_{\rm t}(M,M',t)\, {\rm dM'} = \frac{\sqrt{2/\pi}}{\sigma^2({\rm M'})} \frac{{\rm d}\delta_{\rm c}}{{\rm dt}} \frac{{\rm d}\sigma(M')}{{\rm dM'}} \left[1 - \frac{\sigma^2(M')}{\sigma^2(M)}\right]^{-3/2}$$

$$\times \exp\left\{-\frac{\delta_{\rm c}^2(t)}{2\sigma^2(M')}\left[1 - \frac{\sigma^2(M')}{\sigma^2(M)}\right]\right\}\, {\rm dM'} \qquad (4)$$

is the instantaneous transition rate at t from haloes with M to haloes between M' and $M' + {\rm dM'}$ due to mergers of *whatever amplitude* provided by the original extended PS model (Lacey & Cole 1993). In equation (4), $\delta_{\rm c}(t)$ is the linear extrapolation to the present time t_0 of the critical overdensity of primordial fluctuations collapsing at t, and $\sigma(M) \equiv \sigma(M,t_0)$ is the r.m.s. fluctuation of the density field at t_0 smoothed over spheres of mass M; both $\delta_{\rm c}(t)$ and $\sigma(M)$ depend on cosmology.

The $M(t)$ track followed during accretion by haloes with a given mass $M_{\rm i}$ at a given time $t_{\rm i}$ is therefore the solution of the differential equation

$$\frac{{\rm dM}}{{\rm dt}} = r_{\rm a}[M(t),t] \qquad (5)$$

for the initial conditions $M(t_{\rm i}) = M_{\rm i}$. Strictly speaking, this solution is the *average* track followed by those haloes. Real accretion tracks actually diffuse from it owing to the effects of individual random minor mergers, the scatter remaining nonetheless quite limited along the typical lifetime of haloes (see Raig, González-Casado & Salvador-Solé 2001).

As shown by Raig, González-Casado & Salvador-Solé (2001), the formation of haloes corresponds to rare binary mergers between similarly massive progenitors. Hence, they do cause the rearrangement of the corresponding global systems, the resulting violent relaxation yielding a more or less spherical haloes with new density profiles independent of their past history. In contrast, minor mergers are frequent multiple events, particularly in the case of very small captured haloes. The graininess of accreted matter can therefore be neglected in a first approximation, and the resulting configuration (i.e., a central more or less spherical, relaxed object surrounded by a rather smooth distribution of matter falling into it) can be well approximated by the spherical infall model. Consequently, haloes should grow inside-out during the accretion phase.

3 THEORY VS. SIMULATIONS

The point of view that haloes grow through long periods of gentle accretion after their violent formation in major mergers is clearly much more accurate than the two extreme points of view so far considered in order to derive analytical density profiles (see § 1), namely, the cases of pure spherical infall or one single isolated major merger. During the accretion phase, the inside-out growth condition holds and, since the average mass accretion rate is well-known (eq. 3), one can readily infer the structure developing around the initial seed. On the other hand, the structure of this latter, fixed at formation, should also be possible to determine since the boundary conditions met by the system during violent re-

laxation are well-known, too: they are set by the accretion regime starting at that moment.

3.1 Density profile

Suppose a spherical halo formed at $t_{\rm f}$ with mass $M_{\rm f}$ and radius $R_{\rm f}$. The typical $M(t)$ track followed by the halo during the subsequent accretion phase is therefore the solution of the differential equation (5) with initial condition $M(t_{\rm f}) = M_{\rm f}$. According to the spherical infall model, the accreted mass is deposited, at any moment t, at the instantaneous radius $R(t)$ without altering the inner density profile. Consequently, we have

$$M(t) - M_{\rm f} = \int_{R_{\rm f}}^{R(t)} 4\pi r^2 \rho(r)\, {\rm dr} \qquad (6)$$

with $\rho(r)$ the developing density profile at t. By differentiating this relation taking into account the abovementioned definition of the instantaneous radius

$$R(t) = \left[\frac{3\, M(t)}{f 4\pi \rho_{\rm u}(t)}\right]^{1/3} \qquad (7)$$

and equation (5) we are led to the expression

$$\rho(t) = f\rho_{\rm u}\left\{1 - \frac{M(t)}{r_{\rm a}[M(t),t]} \frac{{\rm d}\ln\rho_{\rm u}}{{\rm dt}}\right\}^{-1} \qquad (8)$$

for the density at $r = R(t)$. Equations (7) and (8) therefore define, in the parametric form, the wanted density profile at any radius $r \geqslant R_{\rm f}$.

But this is not enough to explain the shape of the density profile drawn from simulations. Indeed, a significant fraction of it (of order 50 %) corresponds to the inner region, $r < R_{\rm f}$, fixed at formation (see the position of $R_{\rm f}$ in Fig. 2). To derive the density profile in that inner region we shall take into account that the structure emerging from violent relaxation at $t_{\rm f}$ must satisfy the boundary conditions imposed by accretion starting at that moment. This implies that the inner and outer profiles of any physical quantity must match up at $R_{\rm f}$. Some discontinuity would only appear in case that these boundary conditions were incompatible with the inner structure. But, violent relaxation takes some time to proceed during which surrounding layers begin to be accreted. Hence, the final steady state, far from being incompatible with those boundary conditions, will perfectly adapt to them.

Both $M(t)$ and $\rho_{\rm u}(t)$ entering in equations (7) and (8) are known functions which can be differentiated to any arbitrary order. Consequently, one can derive the values of $\rho(r)$ and any order derivative of it at any radius in the outer region, in particular at the matching radius $R_{\rm f}$. Then, by taking the Taylor series expansion of $\rho(r)$ at $R_{\rm f}$ one can estimate that function at any radius $r < R_{\rm f}$ to any arbitrary accuracy. Although this proves the existence of one unique inner solution, a much practical way to derive it is the following one. As mentioned in § 2, all haloes having the same mass at a given time follow the same accretion track along the common time interval since their respective formation times $t_{\rm f}$, and since the functions $M(t)$ and $\rho_{\rm u}(t)$ coincide for all these haloes in that common time interval, the same is true for the respective outer profiles in the corresponding radial ranges. Moreover, the inner solutions of all

these haloes must coincide with the outer solutions of those having undergone the last major merger early enough since the profiles and derivatives of any order of these latter take identical values at the radii $R_{\rm f}$ of the former and, therefore, have identical Taylor series expansions at such radii. Strictly speaking, this conclusion refers to the *average* density profile since individual profiles will show some scatter around it coming from the abovementioned scatter in real $M(t)$ tracks.

The previous reasoning proves from the mathematical point of view that all haloes with a given mass at a given time have the same (average) density profile regardless of their specific formation time, and that this unique density profile, which naturally appears after the past history of the system has been erased, coincides with the outer solution of haloes formed at an arbitrarily early epoch. From the physical point of view the reason for this apparently surprising result is well understood. As well known, in the case of spherical infall there is only one mass distribution at any moment preserving the steady state of the inner system despite the shell-crossing of the infalling layers; this corresponds to the solution of the spherical infall model for a given accretion rate. In the case of violent relaxation subsequent to a major merger, the resulting steady structure must also adapt to the shell crossing of the infalling layers beginning to be accreted at that moment. This therefore necessarily leads to the same inner mass distribution as in the case of spherical infall at that moment. Note that this explains the result obtained by Huss, Jain & Steinmetz (1999a) that the density profile of haloes closely resembles the NFW profile regardless of whether they form in a hierarchical cosmology or via smooth spherical collapse.

The exact values of f and $\rho_{\rm u}$ used to delimit relaxed haloes in simulations or in the real universe are a mere convention and do not obviously affect their shape. However, from equations (7) and (8) we see that the density profile here predicted depends on the definition of halo radius adopted. There is no contradiction however between these two facts. That definition is crucial, in general, for the ability of the PS model to correctly describe the halo growth process. For instance, the Press-Schechter mass function only fits the empirical mass function provided some specific definition of halo radius as this fixes the mass of objects. Likewise, we cannot pretend to recover the actual density profile of haloes from the PS formalism for any arbitrary definition of halo radius; this must be chosen so to obtain the best agreement between the predictions of the PS model and the results of simulations. Note however that such an "accreting radius" must not necessarily coincide with the arbitrary "delimiting radius" used to compare the resulting theoretical profile with the empirical one obtained by any specific author. In this latter case, we will operate with the theoretical profile previously derived just as if it were the real profile of a simulated (or observed) halo: we shall apply the same delimiting radius which may or may not coincide with the accreting radius previously used. This can be readily done in most cases without any further consideration. Only in the case that the value of f used in the derivation of the profile is larger than the one used to delimit it in the final comparison, or that $\rho_{\rm crit}$ is used in the derivation while $\bar{\rho}$ is used in the comparison some extension of the theoretical profile towards the future will be required. But this sim-
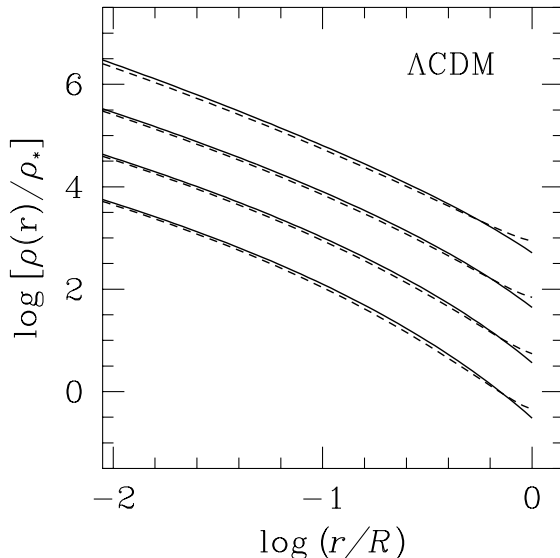

Figure 1. Theoretical density profiles for current haloes of masses $10^{-2}M_*$, $10^{-1}M_*$, M_*, and $10\,M_*$ (from bottom to top) in a flat, $\Omega_{\rm m} = 0.25$, CDM model inferred using $\rho_{\rm u} = \rho_{\rm crit}$ (dashed lines) and $\rho_{\rm u} = \bar{\rho}$ (full lines). In the plot we use the same delimitation of haloes as in Navarro et al. (1996, 1997). M_* is the mass scale at which primordial density fluctuations leave the linear regime, and ρ_* stands for $\rho_{\rm crit}M_*/M$.

ply reflects the real situation met in dealing with simulated (or observed) haloes: their actual steady region extends well beyond any popular delimiting radius.

The correct behaviour of the extended PS model has only been checked in detail in the case of scale-free cosmologies (Lacey & Cole 1994; Raig, González-Casado & Salvador-Solé 2001 for the MPS version). It is therefore necessary to establish the best value of $\rho_{\rm u}$ to be used in case that $\rho_{\rm crit}$ and $\bar{\rho}$ do not coincide. In Figure 1, we plot the theoretical profiles inferred from $\rho_{\rm u} = \rho_{\rm crit}$ and $\rho_{\rm u} = \bar{\rho}$ in a flat, $\Omega_{\rm m} = 0.25$ CDM cosmology. As can be seen, $\rho_{\rm u} = \bar{\rho}$ leads to a decreasing density profile with monotonous varying slope while $\rho_{\rm u} = \rho_{\rm crit}$ leads to a density profile which tends to level off at large radii. (The radii where the two solutions deviate from each other correspond to cosmic times at which Λ becomes non-negligible.) Since such a behaviour is not observed in simulated (or observed) haloes, we conclude that the best value of $\rho_{\rm u}$ to use is $\bar{\rho}$. Concerning the value of f we have checked that the theoretical density profile is always very insensitive to it, any value in the range 178–500 yielding fully indistinguishable results at the resolution of Figure 1. Hereafter, we adopt $f = 200$.

In Figure 2, we compare the average density profiles predicted for current haloes of four different masses in three distinct cosmologies with the corresponding empirical profiles obtained by Navarro, Frenk & White (1997). As can be seen, there is good agreement between theory and simulations, particularly for moderate and low mass haloes. A similar result is found in any cosmology analysed. We want to stress that, once we have fixed the value of the effective threshold $\Delta_{\rm m}$ for major mergers entering in the MPS model, the predicted density profiles of haloes of any mass at any
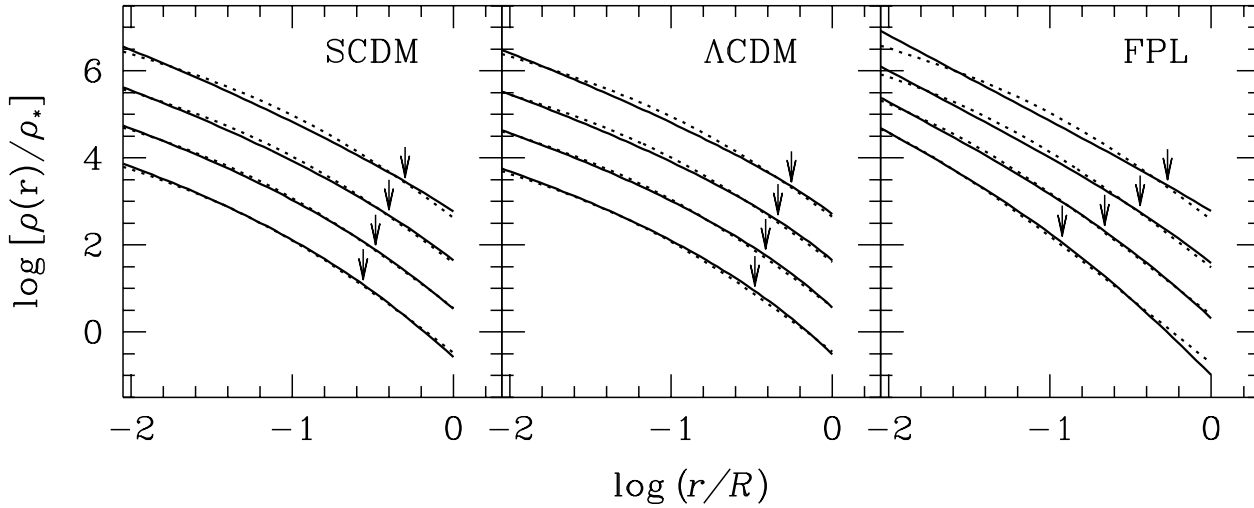
Figure 2. Predicted (full lines) and empirical (dotted lines) density profiles for current haloes with the same masses as in Fig. 1 in the three following cosmologies analysed by Navarro, Frenk & White (1997): a flat, $\Lambda = 0$, CDM model (SCDM), a flat, $\Omega_m = 0.25$, CDM model (ΛCDM), and a flat, $\Lambda = 0$, model with power-law spectrum of density fluctuations of index $n = -1$ (FPL). Arrows mark the frontier between the inner regions fixed at the *typical* formation time and outer ones developed during the subsequent accretion phase.
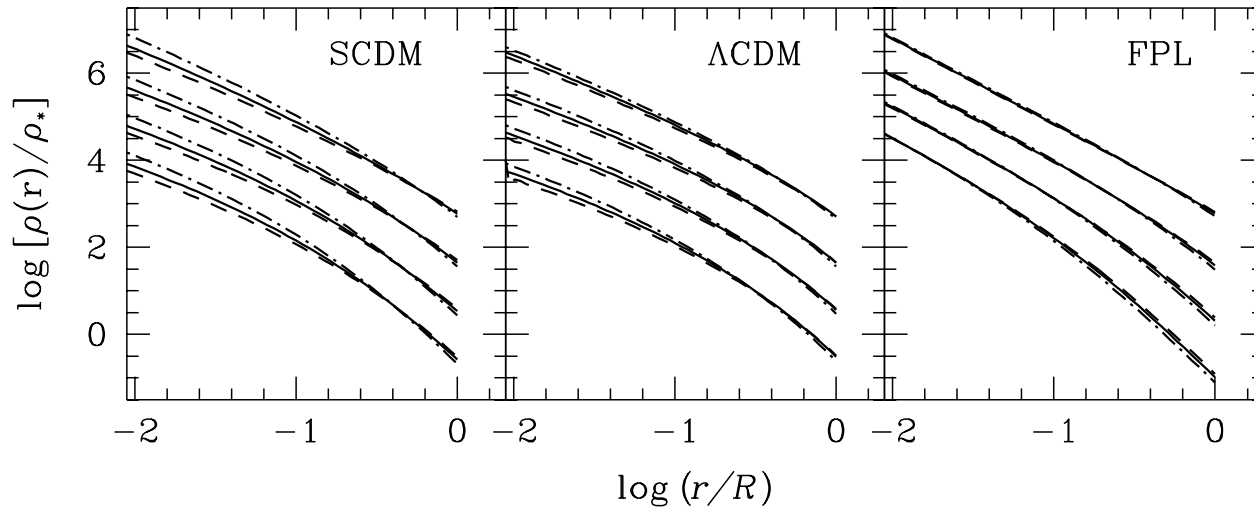

Figure 3. Same density profiles as in Fig. 2 drawn using a value of the threshold for major mergers, Δ_m, equal to 0.5 (full lines) compared to the profiles resulting from $\Delta_m = 0.3$ (dot-dashed lines) and $\Delta_m = 0.7$ (dashed lines).

time are completely determined both in shape and normalization. Thus, there is no freedom to play with. As mentioned in Raig, González-Casado & Salvador-Solé (2001), the comparison between theory and simulations in terms of the description of halo growth alone does not allow us to constrain the value of Δ_m. The only restriction is given a priori by the expected dynamical effects of mergers of different amplitude which points to the plausible range [0.3, 0.7]. In Figure 3 we show the effects of adopting any of these two extreme values of Δ_m. By comparing these results with those obtained from the most reasonable central value of 0.5 shown in Figure 2 we conclude that the value of Δ_m is, on the contrary, well-constrained around that central value by the internal structure of haloes.

The observed trend for the agreement between theory and simulations to degrade towards large halo masses cannot be due to a bad modelling of the halo growth by means of the MPS model. As previously mentioned, the correct behaviour of the mass accretion rate has been checked against numerical simulations in scale-free cosmologies (Raig, González-Casado & Salvador-Solé 2001) such as the one plotted in the right panel of Figure 2 where the departure is the most apparent. To understand the possible origin of this effect we will concentrate on this particular case. In scale-free cosmologies two haloes with different masses, M and M', taken at t and t' satisfying the condition $\sigma(M, t) = \sigma(M', t')$ are indistinguishable. All the density profiles shown in Figure 2 correspond to the present time t_0, but the inside-out growth
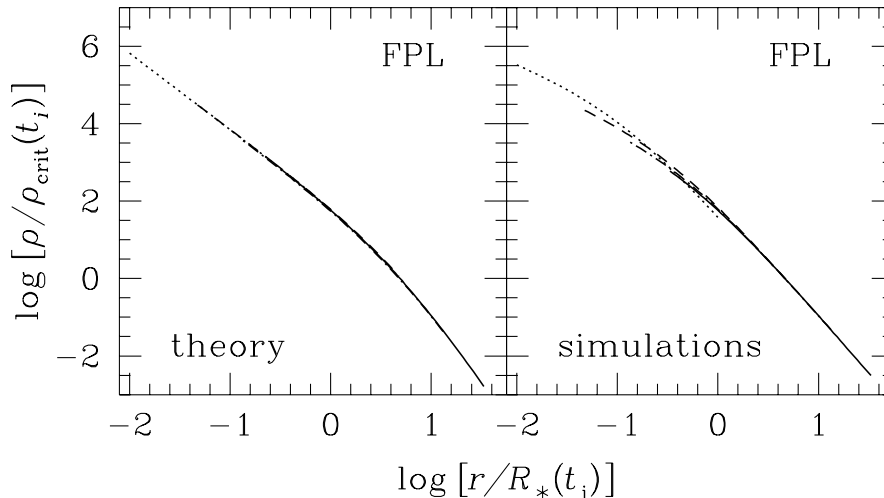
Figure 4. Same density profiles as in the right panel of Fig. 2 but conveniently shifted so to apply to haloes of the four different masses M_i but at four appropriate times t_i making them indistinguishable in that scale-free cosmology (provided the assumption of inside-out growth during accretion). For clarity we have separated the theoretical profiles derived here (left panel) from the empirical ones obtained by Navarro, Frenk & White (1997) identically shifted (right panel). The shifts applied to the profiles at t_0 can be reconstructed from the different symbols used for the non-overlapping empirical profiles: $10^{-2}M_*$ (full line), $10^{-1}M_*$ (dot-dashed line), M_* (dashed line), and $10\ M_*$ (dotted line). $R_*(t)$ is the scale length corresponding to $M_*(t)$.

condition guarantees that, at any other epoch, haloes have the same profiles though truncated at different radii enclosing the corresponding masses. Thus, by conveniently rescaling the density profiles of haloes of different masses at t_0, they must perfectly overlap. In Figure 4 we show the result of applying such shifts on both the four theoretical density profiles shown in the right panel of Figure 2 and the corresponding empirical profiles obtained by Navarro, Frenk & White (1997). (Shifted density profiles are only displayed down to the minimum initial radii where simulated data were fit.) As can be seen, the theoretical density profiles satisfy the expected homologous condition while the empirical profiles do not. There are two possible reasons for this latter fact: either the finite resolution of the simulations introduces some artificial scale breaking the theoretical homologous behaviour of density profiles, or haloes significantly deviate, during accretion, from the inside-out growth implied by the adiabatic invariant collapse approximation. In the former case, one should trust on the theoretical profiles derived here, and conversely in the latter. The fact that the deviation of the empirical profiles from the homologous condition only concerns very small radii where the inside-out growth condition during accretion should only play an indirect role and where the density profiles are the most affected by the finite resolution of the simulations strongly suggests that there may be some problem, indeed, with the empirical profiles there. It would therefore be of great interest to check through numerical simulations of scale-free cosmologies whether or not large mass haloes grow inside-out during accretion as expected from the adiabatic invariant collapse approximation.

The previous discussion is extremely relevant in connection with the problem on the actual slopes of density profiles at very small radii ($r \sim 10^{-2}\ R$). In general, we find good agreement with the slopes reported by Navarro, Frenk & White (1997) (see Fig. 2). But, for large masses in the scale-free case, we find a much steeper slope (as large as -2

in logarithmic units). We want to remark that this is necessary for those density profiles to satisfy the abovementioned homologous condition since the slope at small radii of haloes of a given mass must match the slope at larger radii of more massive haloes. (Note that the asymptotic slope of -2 in the scale-free case is independent, indeed, of the value of Δ_m; see Fig. 3.) If the correct behaviour of the theoretical profiles derived here is confirmed we will be led to the conclusion that the density profile of very massive haloes deviates from the form reported by Navarro and collaborators, and that the asymptotic slope at small radii of the halo density profile actually depends on the particular cosmology considered.

3.2 Velocity dispersion profile

And what about the velocity dispersion profile $\Sigma(r)$? This profile could be readily derived from the Jeans equation (2) using the previous density profile provided that the value of Σ were known at some radius. But it is not. (We are avoiding of course making any unjustified assumption reporting to infinity.) Since, from the dynamical point of view, the unique density profile derived above could perfectly exist side by side with an infinite variety of velocity dispersion profiles the following questions rises. Why does the empirical velocity dispersion profile of haloes show one unique form? What fixes it? As mentioned, in the case that the system has undergone just one unique violent relaxation far in the past and no subsequent accretion of matter we would expect it with a *uniform* velocity dispersion. But this solution is incompatible with the density profile previously calculated imposed by accretion. This suggests the following natural guess: the velocity dispersion profile emerging from violent relaxation at the formation of a halo should be as close as possible to uniform, the only small deviation being forced by the boundary conditions imposed by accretion at that moment.

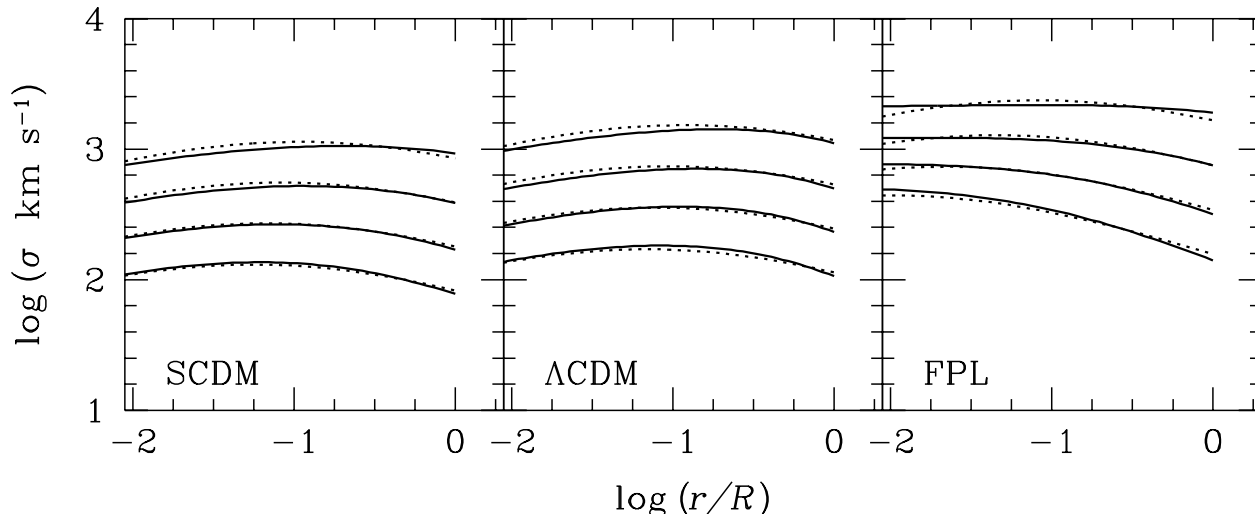To derive such a guessed velocity dispersion profile we

Figure 5. Same as Figure 1 but for the velocity dispersion profile.

will follow an iterative procedure. For the zero-order solution of Σ^2 at $r \leqslant R_f$ we take the square value of the uniform velocity dispersion that violent relaxation tends to establish, given by equation (2) with null logarithmic derivative of Σ^2. Then we compute the zero-order value of Σ^2 corresponding to a slightly larger or smaller, arbitrarily close radius and from the two values we estimate the zero-order logarithmic derivative of Σ^2 at $r \leqslant R_f$. By substituting this derivative into the Jeans equation (2) we obtain the more correct first-order value of Σ^2 at that radius. Finally, by repeating this procedure, we can obtain an estimate as close as wanted to the true value of $\Sigma^2(r)$ at any radius of the inner region. This iterative procedure leads to one unique profile among the infinite number that satisfy the Jeans equation for the density profile derived above using the mass accretion rate. Since we have started the iteration with the values of $\Sigma(r)$ corresponding to an inner uniform velocity dispersion profile (not satisfying that equation), the final solution is necessarily the closest to the latter uniform profile which is, at the same time, compatible with the boundary conditions imposed by accretion as wanted.

Once the value of $\Sigma^2(R_f)$ is known we can use the Jeans equation (2) with the universal density profile derived above to infer the velocity dispersion profile in the outer region. But we can also apply the previous iterative procedure directly to any radius of the outer region. Indeed, the fact that both solutions satisfy the Jeans equation and that they take the same value at R_f guarantees that the two methods yield identical results. Moreover, since haloes formed at different times with initially relaxed regions overlapping over some radial range have, by construction, identical velocity dispersion profiles in the common radial ranges, we are also led to the conclusion that there is one unique velocity dispersion profile for all haloes having the same mass at a given time regardless of their particular formation time, and that this unique velocity dispersion profile coincides with the inner solution, given by the iterative procedure above, of haloes formed at an arbitrarily late time.

In Figure 5 we compare, for the same cosmologies as in Figures 2, the theoretical velocity dispersion pro-

files, inferred using the previous iterative procedure in the whole range of radii, with the empirical profiles obtained by Navarro, Frenk & White (1997). As can be seen the predictions are as good as in the case of the density profiles. This confirms our guess on the origin of the universal velocity dispersion profile.

4   DISCUSSION

It is generally believed that the PS formalism describes the mass growth of dark-matter haloes in hierarchical cosmologies but does not tell us anything about their internal structure. This is however in contradistinction with the idea that such a structure arises just from gravitation (a scale-free force) and *the way that dark-matter haloes grow*. In the present paper, we have shown that, when the distinct dynamical effects of minor and major mergers are taken into account, the PS model also makes definite predictions on the internal structure of haloes, the resulting average theoretical profile being in very good agreement with that drawn from high-resolution numerical simulations. Our results therefore prove that the structure of haloes, including their scaling with mass, is the natural consequence of the combined action of minor and major mergers: outside the total radius of the system at formation the structure is fully determined by the rate at which mass is accreted, while, inside that radius, it results from the initial violent relaxation with boundary conditions set by accretion at that moment. The behaviour of the theoretical profiles derived here strongly suggests that the larger the halo mass, the more apparent are the effects on the respective density profiles of the limited resolution of the simulations. The confirmation of this suspicion would imply a different density profile for very massive haloes from that reported by Navarro, Frenk & White (1996, 1997) and, what is more important, the non-universality of halo structure.

The aim of the present paper was not to derive the accurate distribution of matter in real haloes, but to understand the origin of their empirical apparently universal

structure. For this reason we have considered the simplified case of pure dark-matter haloes as those dealt with in *N*-body experiments. In the real universe, about one tenth of the halo mass is in the dissipative baryonic component, which might have appreciable effects. Likewise, we have assumed spherical symmetry and neglected any rotation of haloes as well as any anisotropy of the velocity tensor while, in hierarchical cosmologies, haloes have a slight angular momentum and, what is more important, they are immersed in large filamentary structures making them accrete matter preferentially along one privileged direction (West 1994) and feel, in their final steady configuration, the tidal field of such anisotropic structures (Salvador-Solé & Solanes 1993). On the other hand, even in the case of exact spherical symmetry, some velocity anisotropy would emerge in the outer part of haloes owing to the distinct evolution of the radial and tangential velocity dispersions in accreted layers during infall. All these secondary effects explain the elongation and slight angular momentum and velocity anisotropy observed in real as well as simulated haloes. (Note however that the density profile derived here is independent of the actual degree of anisotropy of the velocity tensor.) Finally, in the present paper, we have focused on the structure of haloes at $z = 0$, the only redshift for which accurate empirical data for different cosmologies are available (Navarro, Frenk & White 1997). In a forthcoming paper, we will study the predicted dependence on redshift of this structure and compare it with the results of numerical simulations carried out by Bullock et al. (2001) for a ΛCDM cosmology.

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REFERENCES

Bertschinger E., 1985, ApJS, 58, 39
Bond J.R., Cole S., Efstathiou G., Kaiser N., 1991, ApJ, 379, 440
Bower R.J., 1991, MNRAS, 248, 332
Bullock J.S., Kolatt T.S., Sigad Y., Somerville R.S., Kravtsov A.V., Klypin A.A., Primack J.R., Dekel A., 2001, MNRAS, 321, 559
Cohn J.D., Bagla J.S., White M., 2001, MNRAS, 325, 1053
Cole S., Lacey C., 1997, MNRAS, 281, 716.
Del Popolo A., Gambera M., Recami E., Spedicato E., 2000, A&A, 353, 427
Engineer S., Kanekar N., Padmanabhan, T., 2000, MNRAS, 314, 279
Fillmore J.A., Goldreich P., 1984, ApJ, 281, 1
Fukushige T., Makino J., 2001, ApJ, 557, 533
Gott J.R., 1975, ApJ, 201, 296
Gunn J.E., 1977, ApJ, 218, 592
Gunn J.E., Gott, J.R., 1972, ApJ, 176, 1
Hoffman Y., Shaham J., 1985, ApJ, 297, 16
Huss A., Jain B., Steinmetz M., 1999a, ApJ, 517, 64
Huss A., Jain B., Steinmetz M., 1999b, MNRAS, 308, 1011
Jing Y.P., Suto Y., 2000, ApJ, 529, L69
King I.R., 1972, ApJ, 174, L123
Kitayama, T., Suto, Y., 1996, MNRAS, 280, 638
Lacey C., Cole S., 1993, MNRAS, 262, 627.
Lacey C., Cole S., 1994, MNRAS, 271, 676.
Łokas E.L., 2000, MNRAS, 311, 423
Łokas E.L., Hoffman Y., 2000, ApJ, 542, L11
Lynden-Bell D., 1967, MNRAS, 136, 101
Manrique A., Salvador-Solé E., 1996, ApJ, 467, 504
Moore B., Governato F., Quinn T., Stadel J., Lake G., 1998, ApJ, 490, 493
Moore B., Quinn T., Governato F., Stadel J., Lake G., 1999, MNRAS, 310, 1147
Navarro J.F., Frenk C.S., White S.D.M., 1996, ApJ, 462, 563
Navarro J.F., Frenk C.S., White S.D.M., 1997, ApJ, 490, 493
Nusser A., 2001, MNRAS, 325, 1397
Percival W.J., Miller L., Peacock, J.A, 2000, MNRAS, 318, 273
Press W.H., Schechter P., 1974, ApJ, 187, 425
Raig A., González-Casado G., Salvador-Solé E., 1998, ApJ, 508, L129
Raig A., González-Casado G., Salvador-Solé E., 2001, MNRAS, 327, 939
Ryden B.S., 1988, ApJ, 333, 78
Ryden B.S., Gunn J.E., 1987, ApJ, 318, 15
Salvador-Solé E., Solanes J.M., 1993, ApJ, 417, 427
Salvador-Solé E., Solanes J.M., Manrique A., 1998, ApJ 499, 542
Saslaw W.C., 1987, Gravitational Physics of Galactic and Stellar Systems. Cambridge University Press, Cambridge
Shapiro P.R., Iliev I.T., Raga A.C., 1999, MNRAS, 307, 203
Syer D., White S.D.M., 1998, MNRAS, 293, 337
West M.J., 1994, MNRAS, 268, 79
Zaroubi S., Hoffman Y., 1993, ApJ, 416, 410